

ADVANCED PLACEMENT (AP) CALCULUS AB

Grades 11, 12

Unit of Credit: 1 Year

Pre-requisite: Pre-Calculus

Course Overview:

This topic outline is intended to indicate the scope of the course, but is not necessarily the order in which the topics must be taught. Although the AP exam is based on the topics listed here, teachers may wish to enrich their courses with additional topics.

Functions, Graphs, and Limits Analysis of Graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Standard: Limits of functions (including one-sided limits).

- I can intuitively understand the limiting process.
- I can calculate limits using algebra.
- I can estimate limits from graphs or tables of data.

Standard: Asymptotic and unbounded behavior.

- I can understand asymptotes in terms of graphical behavior.
- I can describe asymptotic behavior in terms of limits involving infinity.
- I can compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

Standard: Continuity as a property of functions.

- I can intuitively understand the definition of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)
- I can understand continuity in terms of limits.
- I can understand geometric graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

Derivatives

Standard: Concept of the derivative.

- I can represent graphically, numerically, and analytically the derivative.
- I can interpret the derivative as an instantaneous rate of change.
- I can define the derivative as the limit of the difference quotient.
- I can give examples of how differentiability and continuity are related.

Standard: Derivative at a point.

- I can calculate the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
- I can write an equation of the tangent line to a curve at a point and use the equation to calculate a linear approximation.
- I can calculate the instantaneous rate of change as the limit of the average rate of change.
- I can estimate the approximate rate of change from graphs and tables of values.

Standard: Derivative as a function.

- I can describe the distinguishing characteristics of the graphs of f and f' at corresponding x -values.
- I can describe the relationship between the increasing and decreasing behavior of f and the sign of f' .
- I can state and apply the Mean Value Theorem and its geometric interpretation.
- I can write equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Standard: Second derivatives.

- I can describe corresponding characteristics of the graphs of f , f' , and f'' .
- I can state the relationship between the concavity of f and the sign of f'' .
- I can calculate points of inflection as places where concavity changes.

Standard: Applications of derivatives.

- I can analyze curves, including the notions of monotonicity and concavity.
- I can optimize both absolute (global) and relative (local) extrema.
- I can model rates of change, including related rates problems.
- I can use implicit differentiation to find the derivative of an inverse function.
- I can interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
- I can interpret geometric interpretation of differential equations via direction fields and the relationship between direction fields and solution curves for differential equations.

Standard: Computation of derivatives.

- I can calculate derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
- I can apply derivative rules for sums, products, and quotients of functions.
- I can apply the chain rule and implicit differentiation techniques.

Integrals

Standard: Interpretations and properties of definite integrals.

- I can calculate the definite integral as a limit of Riemann sums.
- I can calculate a definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x) dx = f(b) - f(a).$$

- I can apply basic properties of definite integrals (examples include additivity and linearity).

Standard: Applications of integrals.

Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid of revolution using the disk and washer method, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and accumulated change from a rate of change.

Standard: Fundamental Theorem of Calculus.

- I can use of the Fundamental Theorem to evaluate definite integrals.
- I can use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Standard: Techniques of antidifferentiation.

- I can calculate antiderivatives following directly from the derivatives of basic functions.
- I can evaluate antiderivatives by substitution of variables (including changing the limits for definite integrals).

Standard: Applications of antidifferentiation.

- I can find specific antiderivatives using initial conditions, including applications to motion along a line.
- I can solve separable differential equations and use them in modeling (including the study of the equation $y' = ky$ and exponential growth).

Standard: Numerical approximations to definite integrals

- I can use Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.